Design

The definition of the day is the time it takes for the earth to rotate around its axis so that the sun appears in the same place (with some small differences due to the seasons) however as earth orbits around the sun the time it takes the stars to have an apparent full rotation is about 4 minutes less than a day this is called the sidereal day.

The purpose of this experiment is to produce an estimate for the length of the sidereal day I did this by taking two photos 90 minutes apart as seen in the observations section.

The camera settings and reasons for their choice are as follows:

- Exposure – 60s (allows most light to enter the camera as it is the maximum for the camera)
- Aperture – 3.5 (largest opening allows for most light to enter the camera important when photographing stars)
- Focal length – 14 mm (focused at infinity)
- No flash
- Dimensions 4000 x 3000 pixels
- ISO-100

The equipment is as follows:

- Camera (Micro 4/3 format Panasonic Lumix)
- Tripod
- Time keeping device (I used my phone as it has an accurate time for British summer time)

Observations

Conditions:

Location - 53.880, -0.7063

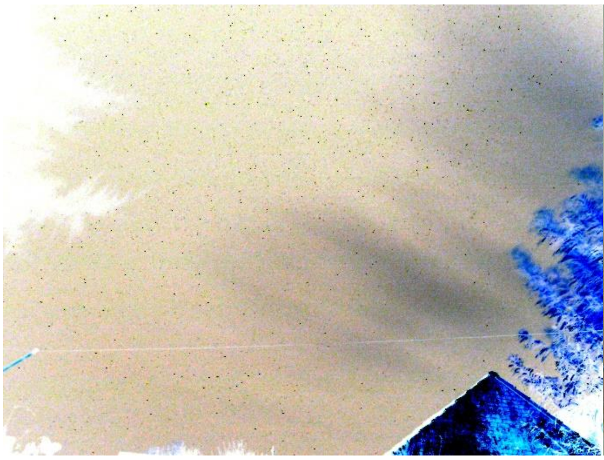
Date - 31/08/22

Time – 22:10, 23:40 BST

Weather – some cloud only affecting first photo

Antoniadi scale – III


The photos were the inverted and had the contrast increased so they could be seen easier.



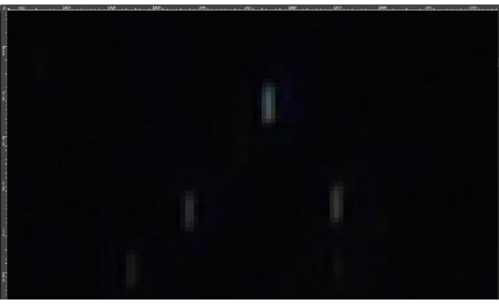

Analysis

I used gimp (open-source photo editing software) to find the coordinates of 18 stars in both photos recording them in a spreadsheet. Using the spreadsheet, I calculated the: distance to the centre of rotation (in pixels); distance between first and second photos (in pixels) and the angle (in radians). More details in appendix 1.

The centre of rotation was not the position of Polaris as it is about two thirds of a degree off the north celestial pole. Thus, the centre of rotation had to be calculated more detail in appendix 2.

The spreadsheet below shows the results and calculations to find the angle for each star and the average angle.

The stars were identified using the online version of Stellarium as seen in the pictures below.


star	first photo x	last photo x	first photo y	last photo y	change in x	change in y	distance from centre x	distance from centre y	distance	arc	angle(rad)
polaris	1354	1361	663	651	7	-12	28.5	24.7	37.71392	13.89244	0.3704789
yildun	1231	1165	439	487	-66	48	-94.63	-199.49	220.7965	81.60882	0.3717479
segin	2825	2916	1085	470	91	-615	1499.37	446.51	1564.443	621.6961	0.4000538
ruchbah	3088	3124	933	209	36	-724	1762.37	294.51	1786.808	724.8945	0.4085273
errai	2025	1888	412	142	-137	-270	699.37	-226.49	735.13	302.7689	0.4148255
epsilon ursea minoris	1025	887	195	339	-138	144	-300.63	-443.49	535.7815	199.4492	0.3744422
zeta ursea minoris	694	548	74	355	-146	281	-631.63	-564.49	847.1159	316.6654	0.3760274
lota cassiopeiae	2534	2675	1207	705	141	-502	1208.37	568.51	1335.426	521.4259	0.3929806
omega cassiopeiae	2532	2600	1008	514	68	-494	1206.37	369.51	1261.692	498.6582	0.3978485
50 cassiopeiae	2310	2373	977	572	63	-405	984.37	338.51	1040.948	409.8707	0.3963364
48 cassiopeiae	2397	2465	998	558	68	-440	1071.37	359.51	1130.08	445.2235	0.3965687
43 cassiopeiae	2591	2633	945	429	42	-516	1265.37	306.51	1301.964	517.7065	0.4003025
psi cassiopeiae	2606	2614	853	333	8	-520	1280.37	214.51	1298.215	520.0615	0.4033256
31 cassiopeiae	2581	2556	763	254	-25	-509	1255.37	124.51	1261.529	509.6136	0.4067633
42 cassiopeiae	2441	2474	912	458	33	-454	1115.37	273.51	1148.415	455.1978	0.399012
40 cassiopeiae	2305	2325	860	462	20	-398	979.37	221.51	1004.108	398.5022	0.3995238
HIP 7078	2475	2488	859	393	13	-466	1149.37	220.51	1170.332	466.1813	0.4010143
HD 19275	2079	2228	1182	861	149	-321	753.37	543.51	928.9615	353.8955	0.3833003
										average	0.3940599

The average angle in radians was calculated to be 0.394 from this I calculated that the sidereal day is 24 hours 5 minutes more detail in appendix 3.

Evaluation

Using the equation below to find the error.

$$\Delta p = p\sqrt{\frac{\Delta T^2}{T} + \frac{\Delta a^2}{a}}$$

Where:

P = sidereal day

ΔP is the error

T is the time

ΔT is the error

a is the measurement of the angle

Δa is the error

ΔT is approximately 3 seconds

Δa is approximately 0.25 degrees

Giving an error of + or – 16.03 minutes which is much larger than the 4 minute difference I am trying to measure. to increase the accuracy, I would try taking longer photos or photos at the same time in two different days. The experiment could also be repeated for more accuracy. An interesting point to note is that the length of the sidereal day can be calculated from the length of the year which using 365.25 days as the length of the year gives 23 hours 56 minutes as the length of the sidereal day which is much more accurate than the figure calculated from the photos.

Appendices

Appendix 1)

Starting by rearranging the cosine rule so that theta is the subject:

$$c^2 = a^2 + b^2 - ab\cos(\theta)$$

$$ab\cos(\theta) = a^2 + b^2 - c^2$$

$$\cos(\theta) = \frac{a^2 + b^2 - c^2}{ab}$$

$$\theta = \cos^{-1}(\frac{a^2 + b^2 - c^2}{ab})$$

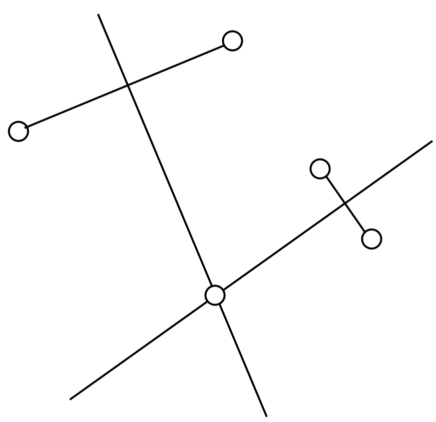
As ABC is a isosceles triangle b can be replaced by a in the equation.

$$\theta = \cos^{-1}(\frac{2a^2 - c^2}{a^2})$$

This equation was used in the spreadsheet in the analysis.

Appendix 2)

The centre of rotation was calculated by first finding the equations for a line connecting the start and end position of two stars (Polaris and Yildun) then finding the equations a line perpendicular going though the midpoint and the finding the intersection of these two lines as can be seen on the diagram below.



$$y = mx + c$$

$$m = \frac{\Delta y}{\Delta x}$$

$$y = \frac{\Delta y}{\Delta x}x + c$$

$$\Delta y = 48$$

$$\Delta x = -66$$

$$y = \frac{-48}{66}x + c$$

$$c = \frac{48}{66}x + y$$

$$x = 1231$$

$$y = 439$$

$$c = \frac{48}{66}1231 + 439$$

$$c = 1334.27$$

$$y = \frac{-48}{66}x + 1334.27$$

$$y = \frac{66}{48}x + c$$

$$c = y - \frac{66}{48}x$$

$$x = 1198$$

$$y = 463$$

$$c = 463 - \frac{66}{48}1198$$

$$c = -1184.25$$

$$y = \frac{66}{48}x - 1184.25$$

The same calculation for the second star:

$$y = mx + c$$

$$\Delta y = -12$$

$$\Delta x = 7$$

$$y = \frac{7}{-12}x + c$$

$$c = \frac{7}{12}x + y$$

$$x = 1354$$

$$y = 663$$

$$c = \frac{7}{12}1354 + 663$$

$$c = 1452.83$$

$$y = \frac{-7}{12}x + 1452.83$$

$$y = \frac{7}{12}x + c$$

$$c = y - \frac{7}{12}x$$

$$x = 1357.5$$

$$y = 657$$

$$c = 657 - \frac{7}{12}1357.5$$

$$c = -134.875$$

$$y = \frac{7}{12}x - 134.875$$

$$y = \frac{66}{48}x - 1184.25$$

$$\frac{7}{12}x - 134.875 = \frac{66}{48}x - 1184.25$$

$$\frac{7}{12}x - 134.875 = \frac{66}{48}x - 1184.25$$

$$0.791666x = 1049.325$$

$$x = 1325.46$$

$$y = \frac{66}{48}x - 1184.25$$

$$y = 638.26$$

$$x = 1325.46$$

Appendix 3)

Rotation In radians:

$$\theta = 0.394$$

Rotation In degrees:

$$\theta = 22.57$$

Rotation Per hour:

$$\theta = 15.05$$

Rotation Per day:

$$\theta = 361.19$$

Rotation per day in hours:

$$\theta = 24.08$$

In hours and minutes:

24 hours 5 minutes